1.

Security	Month 1	2	3	4	5	6
A	3.7%	0.4%	-6.5%	1.4%	6.2%	2.1%
B	10.5%	0.5%	3.7%	1.0%	3.4%	-1.4%
C	1.4%	14.9%	-1.4%	10.8%	4.9%	16.9%

Sample Average (Mean) Monthly Returns

$$\overline{R_A} = \frac{(3.7\% + 0.4\% - 6.5\% + 1.4\% + 6.2\% + 2.1\%)}{6} = 1.22\%$$

$$\overline{R_B} = 2.95\%$$

$$\overline{R_C} = 7.92\%$$

Sample Standard Deviations of Monthly Returns

$$\sigma_A = \sqrt{\frac{(3.7\% - 1.22\%)^2 + (0.4\% - 1.22\%)^2 + (-6.5\% - 1.22\%)^2 + (1.4\% - 1.22\%)^2 + (6.2\% - 1.22\%)^2 + (2.1\% - 1.22\%)^2}{6}}$$

$$= \sqrt{15.34} = 3.92\%$$

$$\sigma_B = \sqrt{14.42} = 3.8\%$$

$$\sigma_C = \sqrt{46.02} = 6.78\%$$

Sample Covariances and Correlation Coefficients of Monthly Returns

$$\sigma_{AB} = \frac{\begin{bmatrix}(3.7\% - 1.22\%) \times (10.5\% - 2.95\%) + (0.4\% - 1.22\%) \times (0.5\% - 2.95\%) + (-6.5\% - 1.22\%) \times (3.7\% - 2.95\%) \\ + (1.4\% - 1.22\%) \times (1.0\% - 2.95\%) + (6.2\% - 1.22\%) \times (3.4\% - 2.95\%) + (2.1\% - 1.22\%) \times (-1.4\% - 2.95\%)\end{bmatrix}}{6}$$

$$= 2.17$$

$$\sigma_{AC} = 7.24; \quad \sigma_{BC} = -19.89$$

$$\rho_{AB} = \frac{2.17}{3.92 \times 3.8} = 0.15$$

$$\rho_{AC} = 0.27; \; \rho_{BC} = -0.77$$

Portfolio Returns and Standard Deviations

Portfolio 1 (X_1 = 1/2; X_2= 1/2; X_3= 0):

$$\overline{R}_{P1} = 1/2 \times 1.22\% + 1/2 \times 2.95\% + 0 \times 7.92\% = 2.09\%$$

$$\sigma_{P1} = \sqrt{(1/2)^2 \times 15.34 + (1/2)^2 \times 14.42 + 0^2 \times 46.02 + 2 \times (1/2 \times 1/2 \times 2.17 + 1/2 \times 0 \times 7.24 + 1/2 \times 0 \times 19.89)}$$
$$= \sqrt{8.53} = 2.92\%$$

Portfolio 2 (X_1 = 1/2; X_2= 0; X_3= 1/2):

$$\overline{R}_{P2} = 4.57\%$$

$$\sigma_{P2} = \sqrt{18.96} = 4.35\%$$

Portfolio 3 (X_1 = 0; X_2= 1/2; X_3= 1/2):

$$\overline{R_{P3}} = 5.44\%$$

$$\sigma_{P3} = \sqrt{5.17} = 2.27\%$$

Portfolio 4 (X_1 = 1/3; X_2= 1/3; X_3= 1/3):

$$\overline{R}_{P4} = 4.03\%$$
$$\sigma_{P4} = \sqrt{6.09} = 2.47\%$$

2.

$\bar{R}_1 = 12\%$	$\bar{R}_2 = 6\%$	$\bar{R}_3 = 14\%$	$\bar{R}_4 = 12\%$		
$\sigma^2_1 = 8$	$\sigma^2_2 = 2$	$\sigma^2_3 = 18$	$\sigma^2_4 = 10.7$		
$\sigma_1 = 2.83\%$	$\sigma_2 = 1.41\%$	$\sigma_3 = 4.24\%$	$\sigma_4 = 3.27\%$		
$\sigma_{12} = -4$	$\sigma_{13} = 12$	$\sigma_{14} = 0$	$\sigma_{23} = -6$	$\sigma_{24} = 0$	$\sigma_{34} = 0$
$\rho_{12} = -1$	$\rho_{13} = 1$	$\rho_{14} = 0$	$\rho_{23} = -1.0$	$\rho_{24} = 0$	$\rho_{34} = 0$

In this problem, we will examine 2-asset portfolios consisting of the following pairs of securities:

Pair	Securities
A	1 and 2
B	1 and 4
C	2 and 3
D	2 and 4
E	3 and 4

We know that the compostion of the GMV portfolio of any two assets *i* and *j* is:

$$X_i^{GMV} = \frac{\sigma_j^2 - \sigma_{ij}}{\sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}}$$

$$X_j^{GMV} = 1 - X_i^{GMV}$$

Pair A (assets 1 and 2):

Applying the above GMV weight formula to Pair A yields the following weights:

$$X_1^{GMV} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = \frac{2-(-4)}{8+2-(2)(-4)} = \frac{6}{18} = \frac{1}{3} \text{ (or 33.33\%)}$$

$$X_2^{GMV} = 1 - X_1^{GMV} = 1 - \frac{1}{3} = \frac{2}{3} \text{ (or 66.67\%)}$$

This in turn gives the following for the GMV portfolio of Pair A:

$$\bar{R}_{GMV} = \frac{1}{3} \times 12\% + \frac{2}{3} \times 6\% = 8\%$$

$$\sigma_{GMV}^2 = \left(\frac{1}{3}\right)^2 (8) + \left(\frac{2}{3}\right)^2 (2) + (2)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(-4) = 0$$

$$\sigma_{GMV} = 0$$

Recalling that $\rho_{12} = -1$, the above result demonstrates the fact that, when two assets are perfectly negatively correlated, the minimum-risk portfolio of those two assets will have zero risk.

For the GMV portfolios of the remaining pairs above we have:

Pair	X_i^{GMV}	X_j^{GMV}	$\bar{R}_{GMV}$	σ_{GMV}
B (i = 1, j = 4)	0.572	0.428	12%	2.14%
C (i = 2, j = 3)	0.75	0.25	8%	0%
D (i = 2, j = 4)	0.8425	0.1575	6.95%	1.3%
E (i = 3, j = 4)	0.3728	0.6272	12.75%	2.59%